Proceedings of the Twenty-eighth (2018) International Ocean and Polar Engineering Conference Sapporo, Japan, June 10-15, 2018 Copyright © 2018 by the International Society of Offshore and Polar Engineers (ISOPE) ISBN 978-1-880653-87-6; ISSN 1098-6189

Modal Analysis of a Top-Tensioned Riser Experiencing Vortex-Induced Vibration and Varying Axial Tensions

Zhe Wang, Di Deng, Decheng Wan^{*}

State Key Laboratory of Ocean Engineering, School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, Collaborative Innovation Center for Advanced Ship and Deep-Sea Exploration, Shanghai, China

*Corresponding Author

ABSTRACT

CFD simulations of a top-tensioned riser experiencing VIV and varying axial tensions are conducted through viv-FOAM-SJTU solver. The varying tension reflects the effect of heave motions of the platform. Strip method is used to simplify the calculation and PimpleDyMFoam module in OpenFOAM is used to solve the fluid field. When the axial tension changes over time, natural frequencies changes over time leading to the changes of VIV response. Internal resonance of the first mode has great impact on the in-line vibration. The multi-modal vibration and modal transitions are illustrated in the simulations.

KEY WORDS: Vortex-induced vibration; viv-FOAM-SJTU solver; strip theory; varying tension; modal analysis

INTRODUCTION

Vortex-Induced Vibration(VIV) of marine risers has been receiving many attentions for decades. The riser is the weakest part in the production system connecting between the platform at the surface and the well at the seabed. VIV is a typical problem of fluid-structure interaction. Vortices generates and sheds alternately from both sides of the riser leading to a periodic vortex-induced pressure around the riser. The riser vibrates under the effect of the pressure and the vibration also affects the flow field around. When the vibration frequency is close to the natural frequency of the riser, "lock-in" phenomenon is usually observed with violent vibration. VIV has become the main source of fatigue damage of the riser. Therefore, it is important to predict the VIV response accurately.

For the VIV problem, many previous works have been done in this field. Some reviews have summarized the research progress in the past decades including computational model, vibration characteristics and mechanisms of the vibration (Wan and Duan, 2017; Williamson and Govardhan, 2004; Wu et al, 2011). Many factors have influences on the VIV response, such as current profiles, aspect ratio and top tensions. Besides, effect of platform motions cannot be ignored, especially for floating structures.

Traditionally, a fixed platform has only small amplitude of motions while a floating structure may have relatively large motions over a long period limited by mooring systems. Effects of the platform motions can be divided into two aspects: motions horizontally and motions vertically. It should be noted that effects of heave motions of the platform are different for different kinds of risers. For a steel catenary riser(SCR), most parts are declining and are not vertical to the surface. Heave motions of the platform generates relatively oscillatory flow between the riser and water. For a top-tensioned riser, it is nearly vertical to the horizontal plane from the surface to the seabed, as shown in Figure 1. The platform relates to the riser by a heave-compensator acting as a spring. The stiffness of the spring is much smaller than the axial stiffness of the riser and the top displacement of the riser is small and can be ignored at a model scale. Therefore, influences of a heaving platform to a top-tensioned riser can be simplified as the influences of the varying axial tensions.

It should be noted that the structural nonlinearity should be focused in the engineering practice, especially for risers with real scale and large aspect ratio. In the real sea conditions, heave displacements of the platform cannot be ignored. Heave motions of the platform can have resonance with horizontal vibrations in both in-line and cross-flow directions for a riser with real scale. This phenomenon has been studied by Chung and Whitney (1981) and examples of coupling of axialbending has been illustrated by Chung and Cheng (1996). For a toptensioned riser with real scale, the pre-tension need to be large enough to keep the riser from buckling at the bottom because of the self-weight. However, the too large pre-tension may cause deformation at the upper part of the riser. This contradiction is one of the reasons that limits the further application of top-tensioned risers in extra deep oceans.

In this paper, as the riser is in model scale, heave resonance and the effect of coupling are ignored. The top tension changes periodically and is a parametric excitation of the riser model. They are quite different problems. Axial varying tensions caused by the heave motion of the platform is the focus in this paper.



Fig. 1 Computational model: (a) refers to the sketch map of the toptensioned riser connected with the platform; (b) refers to the simplified beam model simply supported at both ends.

Under the axial varying tension, VIV of the riser is exacerbated and is plagued with stability problem. Kuiper et al. (2008) made discussion to the parametric instability problem through the Strutt diagram and divided the problem into different cases according to different instability mechanisms. Under extremely large heave motions of the platform, axial tension at the bottom of the riser changes into compression and local buckling may appear. Different mechanisms should be analyzed separately.

Zhang and Tang (2015) performed numerical simulations on the VIV of the top-tensioned riser considering the time-varying axial tension. In the process of "up and down", the heaving platform has larger effect in the "down" period than the "up" period. Under the effect of varying tensions, the stiffness of the riser changes over time and thus, the natural frequency changes over time. Changes of natural characteristics are key problems to analyzing this problem.

Chen et al. (2014) focused the vibration process and built numerical model through finite element model to simulate the VIV of the riser considering the parametric excitation. Results proved that internal resonance existed. The riser model had larger vibration responses under coupled VIV and varying tensions than the cases with only VIV or only varying tensions. Modal transitions appeared from the high mode to the low mode.

Yuan (2018) improved the empirical VIV model originated from Wang (2013) by updating the structural stiffness matrix at each time step. Accuracy of the model was verified by comparing with experiments. Numerical analysis was conducted in time domain. Several new phenomena such as amplitude modulation, time-lag, frequency transition, mode jump and multi-frequencies response superposition are captured in the response comparison with the constant tension case

Previous researches mainly focus on the stability problem, while the study of the vibration process is not enough. Under the heave motions of the platform, VIV of the riser is enlarged leading to severer fatigue damage to the riser. It is important to have deeper understanding on the VIV of a top-tensioned riser with varying axial tensions. As mentioned above, when the axial tension changes over time, natural frequency changes over time. However, the main vibrations mode is determined by natural frequency and Strouhal frequency. An interesting phenomenon appears that the main vibration mode changes over time caused by the varying natural frequency.

Based on the analysis above, the present paper is organized as follows. The following section will introduce the numerical method adopted to solve the VIV problem under the axial varying tensions. Then, the Section PROBLEM will introduce the details of the problem discussed in this paper including computational models and cases. The Section RESULTS followed by Section PROBLEM will compare the differences between the vibration with and without varying tension. The process of "up and down" of the varying tension and modal transition are discussed. Finally, conclusions are made and prospects are proposed.

NUMERICAL METHOD

In this paper, numerical simulations of the VIV and parametric vibration of the top-tensioned riser are conducted by the in-house CFD code, viv-FOAM-SJTU solver. It is developed based on the open source CFD software OpenFOAM.

Computation of Fluid Fields

In the viv-FOAM-SJTU solver, the fluid fields are supposed to be incompressible. Dynamic viscosity μ and the density ρ remains the same during the simulations. The Reynolds-Averaged Navier-Stokes (RANS) equations are solved numerically.

$$\frac{\partial \overline{u}_i}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial}{\partial t} \left(\rho \overline{u}_{i} \right) + \frac{\partial}{\partial x_{j}} \left(\rho \overline{u}_{i} \overline{u}_{j} \right) = -\frac{\partial \overline{p}}{\partial x_{i}} + \frac{\partial}{\partial x_{j}} \left(2\mu \overline{S}_{ij} - \rho \overline{u}_{j} \,' \overline{u}_{i} \,' \right)$$
(2)

where \overline{S}_{ij} refers to the tensor of time-average strain rate and $-\rho \overline{u}_j ' \overline{u}_i '$ refers to the tensor of Reynolds stress.

$$\overline{S}_{ij} = \frac{1}{2} \left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i} \right)$$
(3)

The tensor of Reynolds stress is a new term caused by fluctuation velocity representing the effect of turbulence. SST k- ω turbulence model is introduced to solve the problem.

Equations are solved using finite volume method (FVM) adopting nonstaggered grid which save variable information at the center of the grids. Pimple algorithm is used to decouple the velocity and pressure field. Flow field is solved invoking PimpleDyMFoam solver provided by OpenFOAM. To fit the large displacements of the problem, dynamic mesh module in OpenFOAM is used.

Computation of Structure Fields

Simulation of the riser model is based on Euler-Bernoulli beam theory. The beam model is simply supported at both ends. Dynamic characteristics are simulated through the structural dynamic equations which are a set of second-order ordinary differential equations in two directions (Eq. $4\sim5$).

$$[M]{\ddot{u}_x} + [C]{\dot{u}_x} + [K]{u_x} = {F_x}$$

$$\tag{4}$$

$$[M]\{\dot{u}_{y}\} + [C]\{\dot{u}_{y}\} + [K]\{u_{y}\} = \{F_{y}\}$$
(5)

The model is discretized by finite element method (FEM). In this paper, riser model is divided into eighty units. The unit stiffness matrix K^e has two parts: unit elastic stiffness matrix K^e_E and unit geometry stiffness matrix K^e_G . T refers to the axial tension on the riser and can be expresses as follows.

$$K_{E}^{e} = \frac{EI}{l^{3}} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^{2} & -6l & 2l^{2} \\ -12 & -6l & 12 & -6l \\ 6l & 2l^{2} & -6l & 4l^{2} \end{bmatrix} \qquad K_{G}^{e} = \frac{T}{30l} \begin{bmatrix} 36 & 3l & -36 & 3l \\ 3l & 4l^{2} & -3l & -l^{2} \\ -36 & -3l & 36 & -3l \\ 3l & -l^{2} & -3l & 4l^{2} \end{bmatrix}$$
(6)
$$T(z,t) = T_{t} - \omega_{t}(L-z) + A\sin(2\pi f \cdot t)$$
(7)

The first term in the expression, T_t , refers to the pre-tension. The second term reflects the spatially-varying tension caused by self-weights and buoyancy. The third term refers to the time-varying tension induced by heave motions of the platform.

Calculation of damping adopts the Rayleigh damping model (Clough 2003) as shown in the Eq. 8~9. The coefficients α and β can be calculated by the first two order of natural frequencies f_{n1} and f_{n2} .

$$C = \alpha M + \beta K \tag{8}$$

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \frac{2\varsigma}{f_{n1} + f_{n2}} \begin{bmatrix} 2\pi f_{n1} f_{n2} \\ 1/2\pi \end{bmatrix}$$
(9)

The structural dynamic equations are solved by Newmark-beta method.

Strip Method and Fluid-Structure Interaction

Strip model is applied in the viv-FOAM-SJTU solver instead of threedimensional riser model. Direct simulation of the three-dimensional flow field in time domain through CFD method is sure to be accurate but is hard to achieve and will cost too many resources. Instead, the simplified quasi three-dimensional model, the strip method, is widely used in the simulation of long flexible riser. It is very appropriate for solving CFD simulations of marine risers with high efficiency. Several strips are set to be equally distributed along the riser and each strip contains a two-dimensional circular cylinder. Schematic diagram of the strip method is illustrated in Fig. 2.



Fig. 2 Explanation of the strip model and fluid-structure interaction

In each strip, hydrodynamics of 2D flow around a circular cylinder are calculated through CFD. The fluid forces obtained from the strip can be regarded as forces on the cylinder in the corresponding region. Mapped to the nodes of the structural model, continuous forces distribution on the cylinder are obtained through interpolation and vibration responses are solved by Newmark-beta method. Displacements and velocities of the structure model are transmitted to the corresponding strip to update the flow field and the dynamic mesh. The next time step is ready to be solved.

The strip method is an appropriate way for solving CFD simulations of supramaximal computational domain. It owns high computational efficiency and the computational accuracy is reliable, which has been verified through related researches (Willden and Graham, 2004; Yamamoto et al., 2004). The number of strips is expected to be more than three times of the vibration modes of the riser (Willden and Graham, 2004). The vibration mode can be estimated by calculating the Strouhal frequency and the natural frequencies. The first natural frequency that exceed the Strouhal frequency is approximately the vibration frequency. In this paper, 20 strips are used in the calculation. The whole structure of the viv-FOAM-SJTU solver is shown in Fig. 3.



Fig. 3 Block Diagram of the viv-FOAM-SJTU solver

Modal Analysis

A method of modal analysis is used to decompose the displacements of the riser vibration in both in-line and cross-flow directions following Lie and Kaasen (2006). The displacements can be regarded to be composed by displacements of several modes related to the natural frequencies. Through the modal decomposition, the results can be more intuitive.

At each time step, the equation is established as Eq. 10.

$$y(z,t) = \varphi(z,t)\,\omega(t) \tag{10}$$

where $\omega(t)$ refers to the vector of modal weights and y(z,t) is the vector of modal displacements. $\varphi(z,t)$ represents the matrix of modal shapes. In this paper, the modal weights are solved using the least-square method at each time step.

PROBLEM

The computational model used in this paper follows the experiment of Chaplin et al. (2005). Main parameters are listed in Table 2.

Table 2. Main parameters of the computational model

Parameter	Symbol	Value	Unit
Diameter	D	0.028	m
Length	L	14	m
Mass Ratio	<i>m</i> *	2.4	-
Bending Stiffness	EI	29.88	$N \cdot m^2$
Flow Speed	U	0.4	m/s

Static Top Tension	Tt	1610	Ν
Varying Tension Amplitude	A	0/500	N
Varying Tension Frequency	f	1.1410/3.4367	Hz

Table3. Parameters for cases

Case No.	Pre-Tension (N)	Varying Tension Amplitude (N)	Varying Tension Frequency (Hz)
Case 1	1610	0	0
Case 2	1610	500	1.1410

In the experiment, the riser model is placed in the stepped flow. The lower 45% part is subject to uniform flow while the upper part is in the still water. Layout of the experiment is shown in Fig. 4. Numerical validation following this experiment has been done by Duan et al. (2016) and error of the maximum displacements is within 10%. It should be noted that the riser model is subject to a stepped flow in the experiment while in this paper, the uniform flow is adopted in the numerical simulations.



Fig. 4 Layout of the experiments and results comparisons (Chaplin et al, 2005; Duan et al, 2016)

Validity and reliability of the viv-FOAM-SJTU solver has been verified in several papers (Duan et al, 2016; Fu and Wan, 2017; Fu et al, 2018) and would not be discussed here limited by the length of the paper.

In this paper, effects of the heave motion are reflected as the timevarying axial tension along the riser model. Two cases are used totally in this paper: one case with constant tension as a comparison, and one case with varying tension. The pre-tension is set to be 1610 N.

Twenty strips are set equidistantly along the riser with the same computational domain. Details of the strip model and the initial mesh are shown in Fig. 5. Total number of grids is 42000 and y+ is 2.5. The height of the first layer is 0.000716 meters.



Fig. 5 Strip model and the initial mesh on each strip.

RESULTS

In this section, numerical simulations of the three cases are performed through viv-FOAM-SJTU solver. The calculations are performed on the parallel cluster in Shanghai Jiao Tong University. The time step is set to be 0.0005s to ensure that the maximum Courant number is less than 1.

Natural Characteristics

As mentioned above, modal analysis is used in this paper to distinguish the modal transition during the vibration. Therefore, natural frequencies and the natural modes are needed at first. Under the parameters listed in table 2, the first four-order natural frequencies and natural modes are obtained. The natural frequencies are listed in table 4 and the natural modes are shown in Fig. 6.

Table 4. The first four natural frequencies of the riser model



Fig. 6 The first four natural modes of the riser model

VIV of the Top-Tensioned Riser with Varying Tensions

Numerical results of the two cases are shown in Fig. 7~11. As mentioned in the Section PROBLEM, the first case is a VIV problem with constant tension and is chosen as a comparison. The second case is VIV of a top-tensioned riser with time-varying tension. The frequency of the varying tension is 1.1410Hz, namely the first order natural frequency.

Fig. 7 compares the standard deviation between the two cases in both in-line and cross-flow directions. For the in-line vibration, effect of the varying tension is obvious. The maximum STD displacements is about 0.1 for Case 1 and over 0.6 for Case 2. This can be explained from two aspects. For one reason, the varying tension can be regarded as a kind of "disturbance". The added dynamic energy acquired from the varying tension exacerbates the VIV response of the riser model. As mentioned above, characteristics of the VIV response is correlated to its natural frequencies. Under the effect an axial varying tension, the natural frequencies change over time. Therefore, it can hardly enter the stable state.

For another reason, existence of the internal resonance lead to the differences between the two cases. In the in-line vibration, the riser appears as an arc under the effect of uniform flow, the shape of which is similar to the first mode related to the first order natural frequency. When the tension varies at the frequency of the first order, it inspires

the internal resonances. Therefore, an obvious amplification is observed in the Fig. 7(a).

For the cross-flow vibration, the amplification effect is little, as shown in Fig. 7(b). This is reasonable because there is no internal resonance for the cross-flow vibration.



Fig. 7 Standard Deviation of the displacements: (a) stands for the inline vibration; (b) stands for the cross-flow vibration

Fig. 8~9 show the comparisons of power spectral density of each mode and displacements at several places along the riser model in in-line direction. The frequency component is obtained through FFT algorithm. It should be noted that results in Fig. 8 is calculated by ignoring the first mode. The first mode cannot be shown with other modes in the same figure because the varying tension dominates the in-line vibration as explained in Fig. 7. If all the modes are shown in the same figure, no frequency component except 1.1410 Hz appears in the figure.

Under the effect of the varying tension, multi-modal vibrations are inspired. Previous researches have shown that a long flexible cylinder tends to vibrate at various modes (Willden and Graham, 2004; Vandiver et al, (2009)). The varying tension inspires the multi-modal vibration of the riser model. The frequency component at the third mode equals to the frequency of the varying tension.

Fig. 9 shows the power spectral density figure of the in-line vibration at different places along the riser model. Fig. 9(b) shows that the frequency component of the varying tension appears mainly at the middle parts of the riser while in Fig. 9(a), results is consistent with the vibration mode.



Fig. 8 Comparison of modal power spectral density between case 1 and

case 2 in the in-line direction.



Fig. 9 Comparison of power spectral density between case 1 and case 2 in the in-line direction.

Fig. 10~11 show the comparisons in cross-flow direction. Effects of the varying tension are not the same as the in-line vibration. The sub-harmonic vibration appears in the first mode (Fig. 10) and in the middle part of the riser (Fig. 11) which is a typical phenomenon in the parametric excitations. It reflects the nonlinearity of the system.

Similar to the in-line vibration, more frequency components appear in the power spectral density figure. However, it is different that the varying tension do not dominate the vibration frequency. The original VIV frequency components are still obvious.

Another phenomenon that should be noticed is the width of the frequency component. In Fig. 11(a), the frequency of VIV is obvious with only one peak. In Fig. 11(b), except for the sub-harmonic components in the middle part of the riser, the original VIV components become wider with several peaks. One possible explanation lies on the continuous varying tension. When the axial tension is constant, the vortex-induced vibration is stable related to the axial tension. When the tension changes continuous over time, the related frequency component changes. Therefore, a wider response spectrum is obtained.



Fig. 10 Comparison of modal power spectral density between case 1 and case 2 in the cross-flow direction.



Fig. 11 Comparison of power spectral density between case 1 and case 2 in the cross-flow direction.

Modal Transition

In this section, the phenomenon of modal transition is recognized through the wavelet transformation (Fig. 12) and the spatial-temporal contour (Fig. 14).

In the previous section, numerical results obtained by FFT are discussed in detail. Disadvantages of the results are obvious that they can only reflect the frequency component from the data. The vibration frequency at each time cannot be reflected. Therefore, the wavelet transformation is used to recognize the vibration frequency and the vibration mode at each time.

As shown in Fig. 12(a), the wavelet contour reflects the vibration frequency component at each time step. To be clearer, the main vibration frequency at each time is picked up and is shown in Fig. 12(b). The first four order natural frequencies are marked on the figure. From the Fig. 12, conclusions can be made that the vibration covers the first four order natural frequencies.

At any time, if the energy covers more than one order of the natural frequency, modal transition happens at that time. Four typical time is chosen to illustrate the process of modal transition as shown in Fig. 13. Four typical time is: $23 \sim 25$ s, $43 \sim 45$ s, $52 \sim 54$ s and $57 \sim 59$ s. At first three time, energies in Fig. 12(a) cover more than one natural frequencies, while at fourth time, the energy only cover the third mode natural frequency. A clear third mode is shown in the fourth figure in Fig. 13.



Fig. 12 Wavelet contour and the main vibration frequency in the cross-flow direction.



Fig. 13 Four typical time chosen to illustrate the process of the modal transition.

Besides the wavelet transformation, the spatial-temporal contour is more convenient and intuitive to show the modal transitions. The contour within 23.5~25.25 s is shown in Fig. 14. It reflects the vibration shapes at each time and how the vibration evolves. The horizontal coordinate refers to time and the vertical coordinate represents the length along the riser model. At the time right after 23.5 s, the riser vibrates at the fifth mode and it changes into fourth mode before 24 s. After 24 s, the main vibration mode changes into the third mode and standing waves appears in the contour after 24.5 s which is related to the "lock-in" phenomenon of vortex-induced vibration.



Fig. 14 Spatial-temporal contour within 23.5~25.25 s of cross-flow vibration.

Noticed that in this section, analysis above is made based on the results of cross-flow vibration shown in Fig. 12~14. Reason is that as shown in the last section, the vibration frequency is dominated by the varying tension (Fig. 9(b)). The phenomenon of modal transition mainly happens at cross-flow vibration. Wavelet contour of the in-line vibration is shown in Fig. 15. The varying tension dominates the vibration frequency powerfully.



Fig. 15 Wavelet contour of the in-line vibration

Periodic Vibration

In this section, the process of varying tension is shown with spatialtemporal contour. The vibration behavior is analyzed in time-domain. Fig. 16 shows the contour of in-line vibration from 20.0 s to 23.5 s. The axial tension varies at the frequency of the first order natural frequency and is relatively slow related to the system. The periodic vibration response is obvious in the in-line vibration. When the tension turns to larger and smaller, displacements of the in-line vibration change accordingly. The effect of reduction process is larger than the enlargement process.



Fig. 16 Spatial-temporal contour of the in line-vibration in Case2

Fig. 17 shows the contour of the cross-flow vibration from 23.5 s to 27.0 s. The time history curve is shown together with the contour. Three kinds of typical vibration response appear in this period including modal transition, stationary wave, and the travelling wave. The problem of modal transition has been discussed in the previous section. The stationary wave usually represents that "lock-in" happens, while the travelling wave is quite common for flexible riser with large aspect ratio.



Fig. 17 Spatial-temporal contour of the cross-flow vibration in Case2

CONCLUSIONS

In this paper, numerical simulations of a top-tensioned riser experiencing vortex-induced vibration and varying axial tension is conducted by viv-FOAM-SJTU solver. Numerical methods adopted in the solver are introduced briefly. For a top-tensioned riser, the effect of heave motion of the platform can be regarded as a time-varying axial tension. When the axial tension changes over time, the bending stiffness changes over time leading to the changes of natural frequencies. Since most of the vibration characteristics are related to the natural characteristics, several changes happen. This is the core of this problem.

When the frequency of varying tension equals to the first order natural frequency, internal resonance happens and the VIV response is exacerbated. Under the effects of varying tensions, more frequency components appear in the vibration. The multi-modal vibration is inspired and the modal transition happens. Through the comparisons in Fig. 8~11, different frequency components can be distinguished. The traditionally multi-modal vibration for flexible risers is different from the modal transition phenomenon in the case of varying tension.

The process of modal transition is shown through wavelet transformation and spatial-temporal contour. Modal transition is obvious in the cross-flow vibration, while in the in-line vibration, the varying tension dominates the vibration frequency. Under the effect of the varying tension with low frequency, periodic vibration appears in the in-line direction (Fig. $15\sim16$).

Admittedly, the present work has considerable space to improve. Realization of the CFD simulation of VIV and effects of platform motions are the focus in the completed work. Up to now, numerical simulations are performed under a model scale. Risers with real scale can be explored by our solver in the future. Besides, effects of axial resonance are expected to be considered in the following papers.

ACKNOWLEDGEMENTS

This work is supported by the National Natural Science Foundation of China (51490675, 11432009, 51579145), Chang Jiang Scholars Program (T2014099), Shanghai Excellent Academic Leaders Program (17XD1402300), Program for Professor of Special Appointment (Eastern Scholar) at Shanghai Institutions of Higher Learning (2013022), Innovative Special Project of Numerical Tank of Ministry of Industry and Information Technology of China (2016-23/09) and Lloyd's Register Foundation for doctoral student, to which the authors are most grateful.

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